MICROWAVE, DOPPLER INVARIANT, PULSE COMPRESSION FILTERS

by

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Abstract

The complete design theory, including explicit formulae for element values, is presented for waveguide or stripline pulse compression filters which approximate an ideal response in an optimum maximally flat manner. For an nth degree filter, peak power compression ratios in excess of n are shown to be possible.

Summary

Due to the properties of electromagnetic waves being reflected from metallic objects moving with constant velocities, it may be shown that the Doppler invariant signal satisfies the equation

$$\gamma S(\gamma t) = e^{jK_o} S(t - t_o) \qquad (1)$$

where K_o and t_o are functions of

$$\gamma = \frac{1 + \Delta^2 + 2\Delta \cos_{o}}{1 - \Delta^2}$$

where

$$\Delta = \frac{V}{C}$$

θ_o = angle of incidence

and V and C are the velocities of the target and light respectively. One solution to equation (1) is a frequency modulated signal of the form,

$$S(t) = A\left(1 - \frac{t}{K}\right)^{\frac{jK}{\alpha} - 1} e^{\frac{jt}{\alpha}} \qquad (2)$$

which is applied in the time interval $-T \leq t \leq 0$ and where the carrier has been suppressed.

The frequency spectrum of this pulse is given approximately by

$$S(j\omega) = A(1 - \alpha\omega)^{\frac{jK}{\alpha}} \quad 0 < \omega < \omega_1 \qquad (3)$$

$$= 0 \quad \omega < 0,\ \omega > \omega_1$$

where

$$\omega_1 = \frac{T}{\alpha(K + T)}$$

The output signal from the matched filter for this signal is

$$A\sqrt{\frac{\alpha K}{2\pi}}\,\frac{\sin\left[\omega_1(t - K)\right]}{(t - K)} \qquad (4)$$

which is an unmodulated signal with a $\frac{\sin x}{x}$ time domain envelope.

The Matched Filter

From equation (3) the frequency response of the ideal matched filter in normalized form is given by

$$S(p) = (1 + j\alpha p)^{\frac{j2}{\alpha}} \qquad |\omega| < \omega_c \qquad (5)$$

$$= 0 \quad |\omega| > \omega_c$$

where $\omega_c = \frac{\omega_1}{2}$ and the mid-band delay is 2.

It may be shown that this response may be approximated in an optimum maximally flat manner by the rational transfer function

$$S_{12}(p) = \frac{(2n - 1)P_n^{*(\beta)}(p) + p(1 - jn\beta)P_{n-1}^{*(\beta)}(p)}{(2n - 1)P_n^{(\beta)}(p) + p(1 - jn\beta)P_{n-1}^{(\beta)}(p)} \qquad (6)$$

where the polynomial $P_n^{(\beta)}(p)$ is defined as

$$P_n^{(\beta)}(p) = \frac{(1 + j2\beta p)^{\frac{-j}{\beta}} p^{2n+1} \prod_{r=1}^{n} (1 + (r\beta)^2)}{(2n)!} . I_n^{(\beta)}(p)$$

where

$$I_n^{(\beta)}(p) = \int_1^{\infty} \left[1 + j(1 + x)\beta p\right]^{\frac{j}{\beta} - n - 1} (x^2 - 1)^n dx \qquad (7)$$

with $\beta = \frac{\alpha}{2}$.

Alternatively, $P_n^{(\beta)}(p)$ may readily be constructed using the recurrence formula,

$$P_{n+1}^{(\beta)}(p) = (1 + j\beta p)P_n^{(\beta)}(p) + \frac{p^2\left[1 + (n\beta)^2\right]}{(2n - 1)(2n + 1)} . P_{n-1}^{(\beta)}(p) \qquad (8)$$

with the initial conditions

$P_o^{(\beta)}(p) = 1,\ P_1^{(\beta)}(p) = 1 + (1 + j\beta)p$

The scattering transfer coefficient $S_{12}(p)$ may be realized between ports one and two of the non-reciprocal network shown in Fig. 1 where the third port of the circulator is connected to resistively terminated loss-less two-port. From the recurrence formula (8), it may be shown that this two-port is in the form of shunt inductors separated by unity impedance ideal phase shifters as shown in Fig. 2, where

$$\frac{1}{L_r} = (2r - 1)$$

and (9)

$$\theta_{r,r+1} = -\cot^{-1}(r\beta)$$

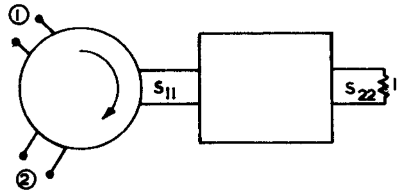

FIG. 1.

Microwave Realizations

By applying the frequency transformation

$$\omega \rightarrow B(1 - \frac{\gamma_g}{\gamma_{go}}) \qquad (10)$$

the waveguide pulse compression filter shown in Fig. 3 may be directly obtained from the element values of the low-pass prototype. Typically, the X-band, pulse compression factors of 50→100 may readily be obtained with range resolutions of less than one metre. Strip-line or microstrip filters may readily be designed in a similar manner using resonators loosely coupled to a single uniform line.

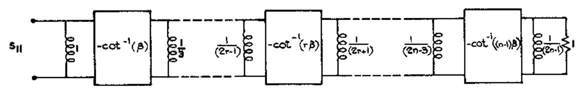

FIG. 2

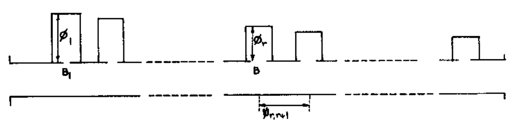

FIG 3